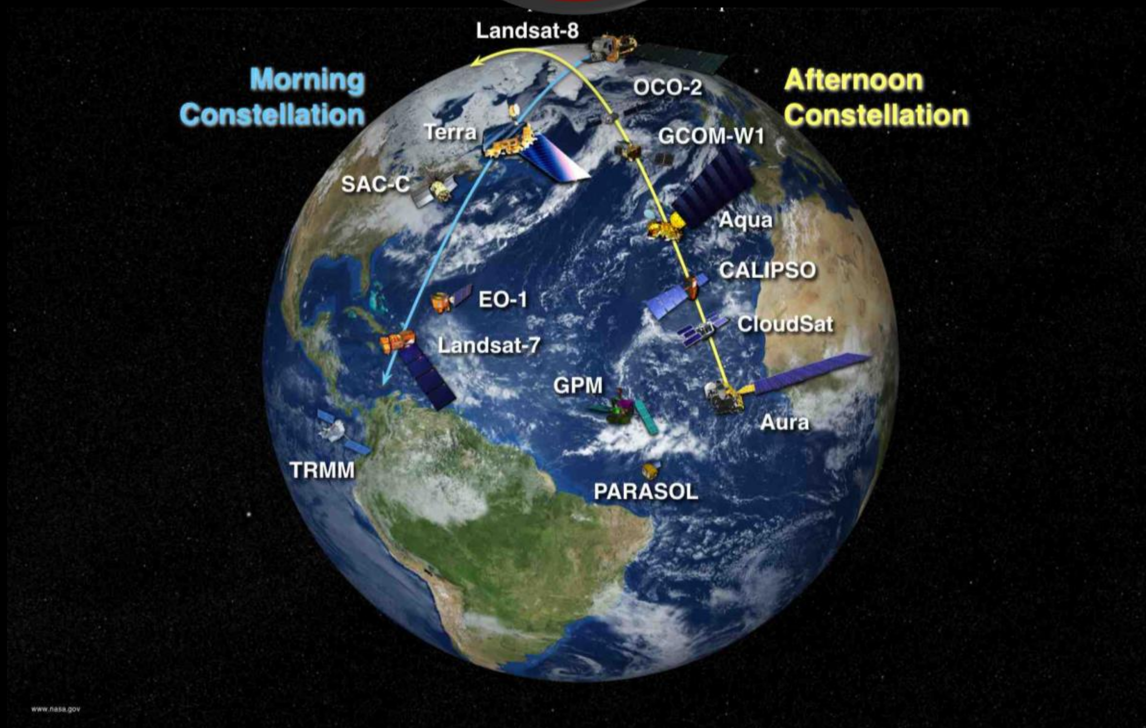

United States Brazil Canada Finland France Japan Netherlands United Kingdom

International Earth Science Constellation Mission Operations Working Group

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Earth Observation System Flight Dynamics System Covariance Realism

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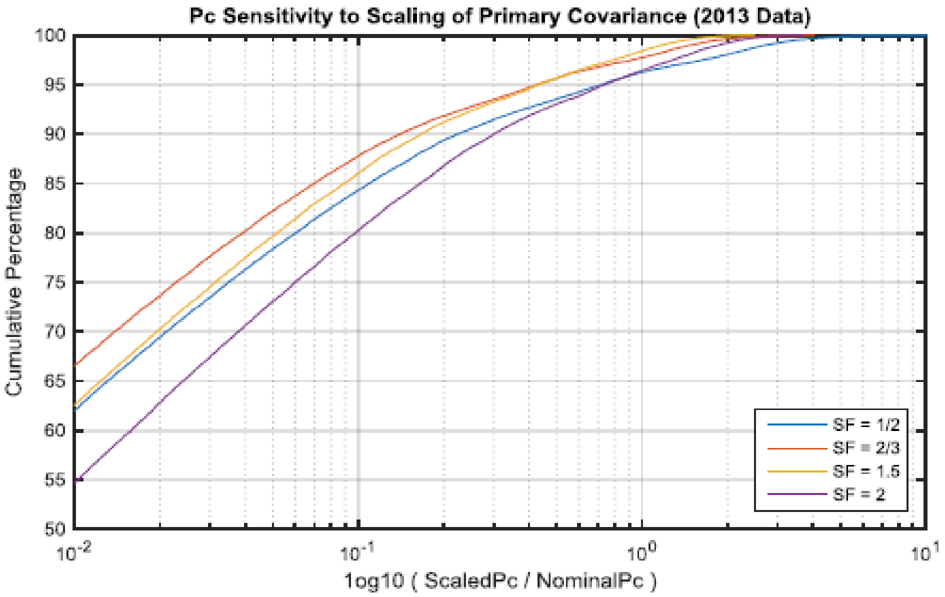
Motivation

- At its User’s Forum on 14 Apr 2015, CARA recommended its users to begin delivering realistic covariances.
- This presentation is a response to that recommendation.
- Aqua and Aura’s covariances have been tuned during times without maneuvers.
- The impact on the probability of collision (on select conjunctions) using a tuned covariance was examined.
- A method to tune covariances through maneuvers is being adopted and will be ready for presentation by the next MOWG.

Introduction

- **Covariance Realism:**
  - Study the evolution of a set of covariances (propagated into the future beginning with a pre-determined definitive state estimate error) by examining its behavior at equally spaced propagation points.
  - Uses *inferential statistics* in which behavioral conclusions for a large population are drawn using sample data.
  - The Mahalanobis distance of a covariance at a particular propagation point represents the ratio of the predicted minus definitive position difference to the covariance's prediction.
  - A group of the squares of such calculations should conform to a chi-squared distribution with 3 Degrees-of-Freedom (DoF)
- **Involves the following 3 phases:**
  - Collection/calculation of definitive state estimates through orbit determination.
  - Calculation of covariance realism test statistics at each propagation point.
  - Proper assessment of those test statistics using a hypothesized distribution.

- **P_c sensitive to Scaling of Primary Covariance:**
  - Graph below was presented at the 14 Apr 2015 CARA User's Forum.
  - Depicts P_c differences between nominal value and recalculation with primary covariance rescaled (Scale Factors 0.5 to 2).



- **~2–5% of cases show Scaled P_c is greater than the Nominal P_c.**
  - Impacts operational conclusions.
- **A realistic covariance is important.**

Definition of Terms

Definitive State Estimate:

- Best known position and velocity at an epoch time; obtained by passing observations through a Filter or Batch estimator.

Definitive State Estimate Error:

- Uncertainty in the definitive state estimate produced by a Filter or Batch estimator.
- Contained in a Definitive Covariance Matrix.

Predicted State Estimate:

- Position and velocity are propagated to a time t using a state transition matrix and definitive state estimate at an epoch time t_0.

Predicted State Estimate Error:

- Uncertainty in the Predicted State Estimate propagated using a force model.
- Contained in a Predictive Covariance Matrix.

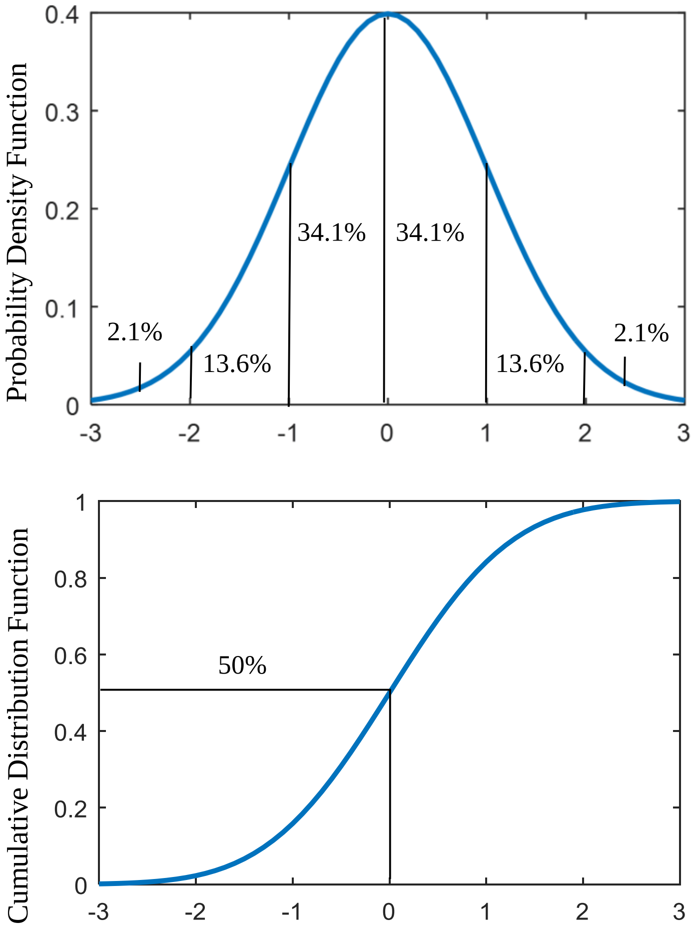
Epoch Covariance:

- State Estimate Error at a specific epoch.

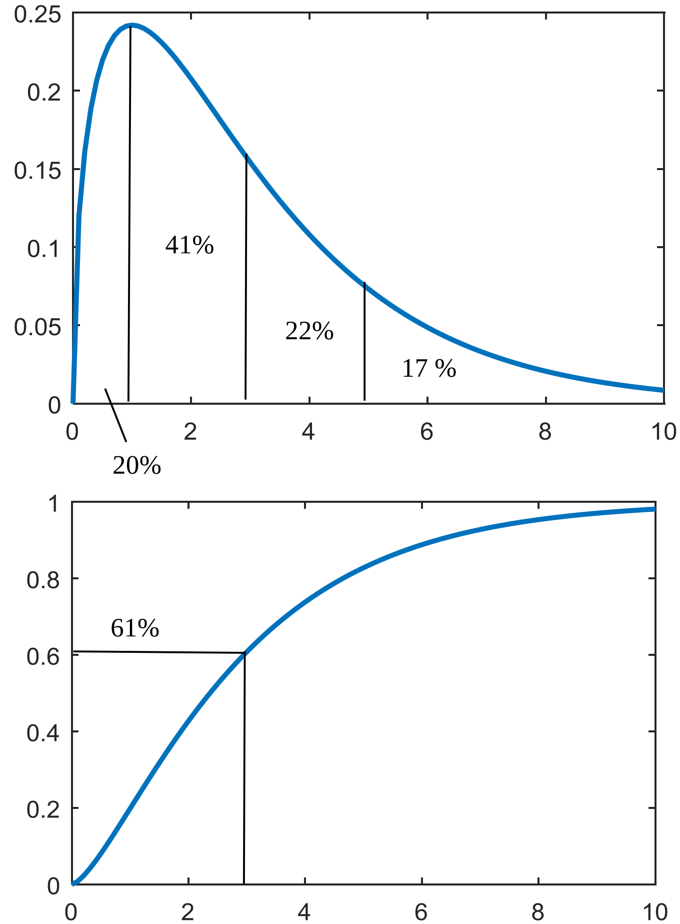
Predicted – Definitive State Estimate:

- The difference between the predicted state estimate (propagated from epoch time t_0) and the definitive state estimate (obtained through orbit determination) at any time t.

Gaussian versus 3 DOF Chi-Square Distribution



Normal Distribution



3 DOF Chi-Square Distribution

- A Normal or Gaussian Distribution has a Mean of 0 and Standard Deviation of 1:
  - 50% of values are distributed above and below a Mean of 0
- A Chi-Square Distribution is a multi-variate distribution of the sum of the squares of *k* independent standard normal random variables.
- A *k* degree-of-freedom (DOF) Chi-Square Distribution has a mean value of *k*.
- A Chi-Square Distribution with 3 DOF has a Mean of 3 and a Standard Deviation of 8/3:
  - 61% of values are distributed below a Mean of 3

Chi-Square Statistic

- **The Chi-Square statistic is computed using the vector of predicted – definitive state estimates, *ε*, and the predicted state estimate error or covariance matrix, *C*:**

$$\chi^2_{3dof} = \varepsilon C^{-1} \varepsilon^T \approx \left(\frac{\varepsilon_R}{\sigma_R}\right)^2 + \left(\frac{\varepsilon_I}{\sigma_I}\right)^2 + \left(\frac{\varepsilon_C}{\sigma_C}\right)^2 \qquad \varepsilon = \begin{bmatrix} R_{predicted} - R_{definitive} \\ I_{predicted} - I_{definitive} \\ C_{predicted} - C_{definitive} \end{bmatrix}$$

Inverse of the covariance matrix obtained directly from propagation, thus containing correlation terms

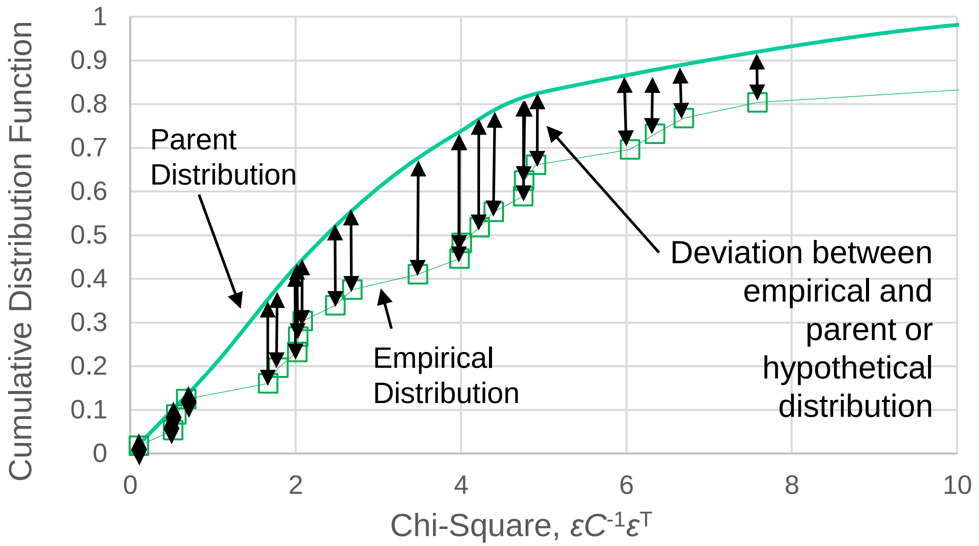
Predicted State Estimate Error assuming no correlation between variables

Difference between predicted and definitive state estimates

- **A perfectly sized covariance should have a Chi-Square equal to 3.**
- **In fact, this first moment of distribution test provides an initial idea of a covariance's departure from reality.**
- **However, a more rigorous Empirical Cumulative Distribution Function (ECDF) Method is adopted for this covariance realism analysis.**

Quadratic Statistics[1]:

- An ECDF method that evaluates how well an empirical distribution corresponds to a parent distribution by examining the summation of a function of the squares of the deviations between the empirical and parent distributions.
- Examples are the Cramér – von Mises, Watson, and Anderson-Darling statistics.
- This analysis uses the more permissive Cramér – von Mises statistic due to the likelihood of outliers.



**Note in this example the Parent Distribution is a 3 DOF Chi-Square Distribution

P-value and Confidence Interval:

- *P*-value: The likelihood an empirical distribution is drawn from a parent distribution.
- Confidence Interval: A *p*-value threshold that indicates a "pass" or "fail". Normally 2% or higher are accepted.

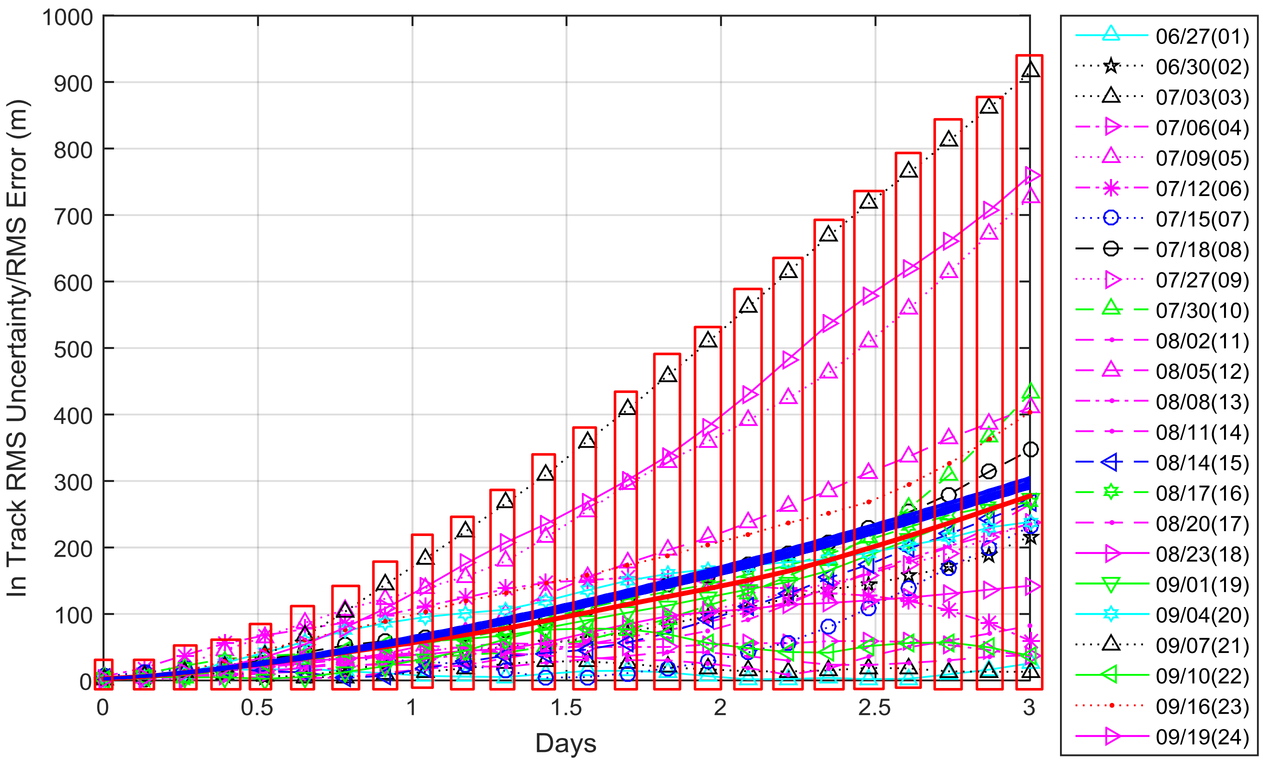
$$Q = n\int_{-\infty}^{\infty}\left[F_n(x) - F(x)\right]^2 \psi(x)dx$$

EDF Statistic
Empirical CDF
Hypothetical CDF
Weighting Function

A *p*-value can be obtained for each *Q*-Statistic using a published table of *p*-values; one that is generated using Monte Carlo simulations.

Chi-Square Statistic Grouping

- Collect bins of Chi-Square Statistics at each propagation point and examine their Chi-Square distribution .
- The number of Chi-Square Statistics in each bin should be equal to the number of total propagations.



Each red rectangle or "bin" here contains information of a set of covariances at a unique propagation point.

Process Noise Effect and Implementation

- State Noise Compensation (SNC) - process noise is added to the propagation of the definitive state estimate in order to account for uncertainty in the force model.

- The predicted state estimate error, $P(t)$, is propagated using linear mapping as follows: $P(t) = \varphi P(t_0)\varphi^T + \Gamma Q \Gamma^T$

  φ, Γ = state transition matrices

  $P(t_0)$ = definitive state estimate error

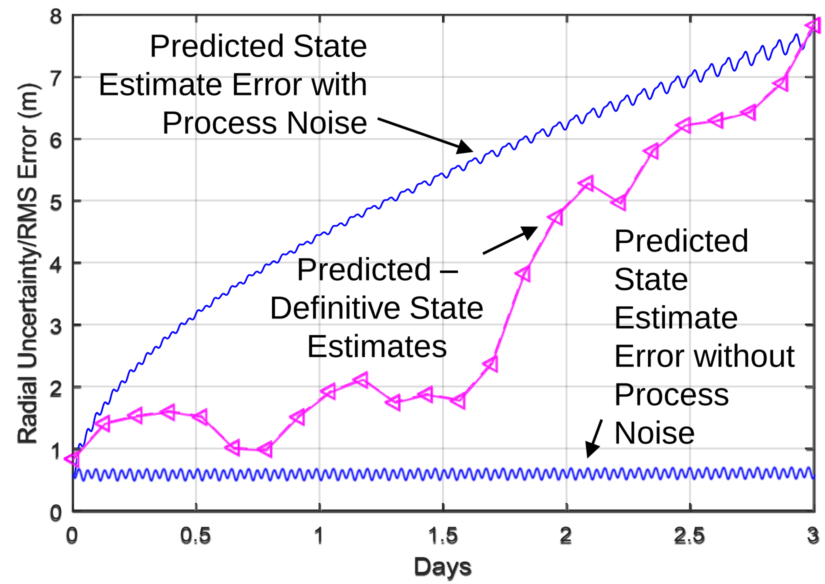
- The process noise matrix Q is built using variances in acceleration as follows:

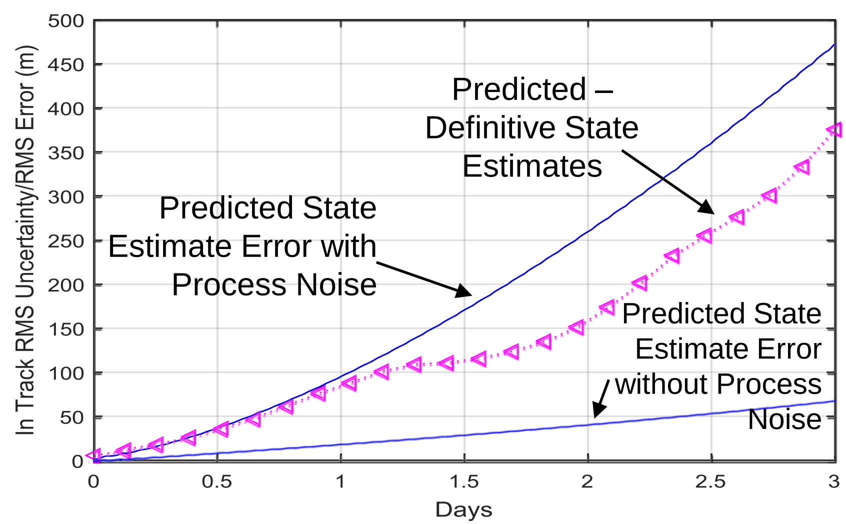
$$Q_{RIC} = \begin{pmatrix} \frac{\Delta T^4}{3} I \cdot \bar{q}_{acc} & \frac{\Delta T^3}{2} I \cdot \bar{q}_{acc} \\ \frac{\Delta T^3}{2} I \cdot \bar{q}_{acc} & \Delta T^2 I \cdot \bar{q}_{acc} \end{pmatrix} \qquad \bar{q}_{acc} = \begin{pmatrix} \sigma_{\ddot{R}} \\ \sigma_{\ddot{I}} \\ \sigma_{\ddot{C}} \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

ΔT = propagation step size

$\sigma_{R,I,C}$ = acceleration variances





- **The following assumptions are made in the study:**
  - Propagation Date Span: 2 Aug 2014 to 6 Nov 2014.
  - Propagation Time Span: 3 Days.
  - Maneuvers occur on 27 Aug, 17 Sep, 8 Oct, and 21 Oct 2014 – propagation over these dates are avoided.
  - Process noise is kept constant for all propagations.
- **The study is conducted as follows:**
  - Select an arbitrary set of acceleration variances and propagate all definitive state estimates using the corresponding process noise.
  - Examine the deviation between the ECDF and CDF of the 3 DOF Chi-Square Distribution without outlier identification.
  - Perform an outlier identification test and eliminate propagations that contain outliers.
  - Resize (by adjusting the process noise) the predicted state estimate error using the total mean RMS error of all remaining propagations (after outlier identification).
  - Examine the deviation between the ECDF and CDF of the 3 DOF Chi-Square Distribution (post outlier identification and resized predicted state estimate error).

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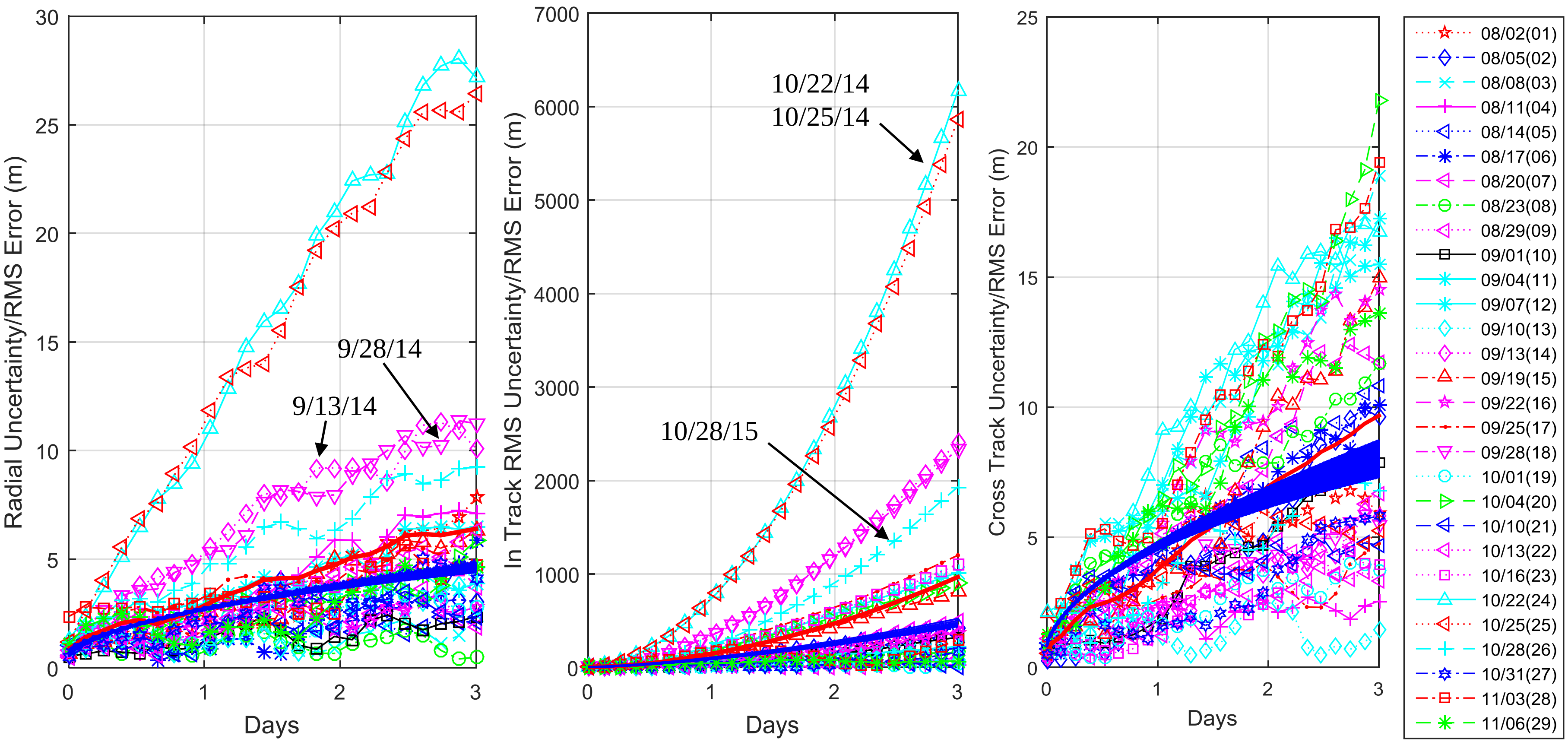
RMS Error and Uncertainty

(Without Outlier Identification)

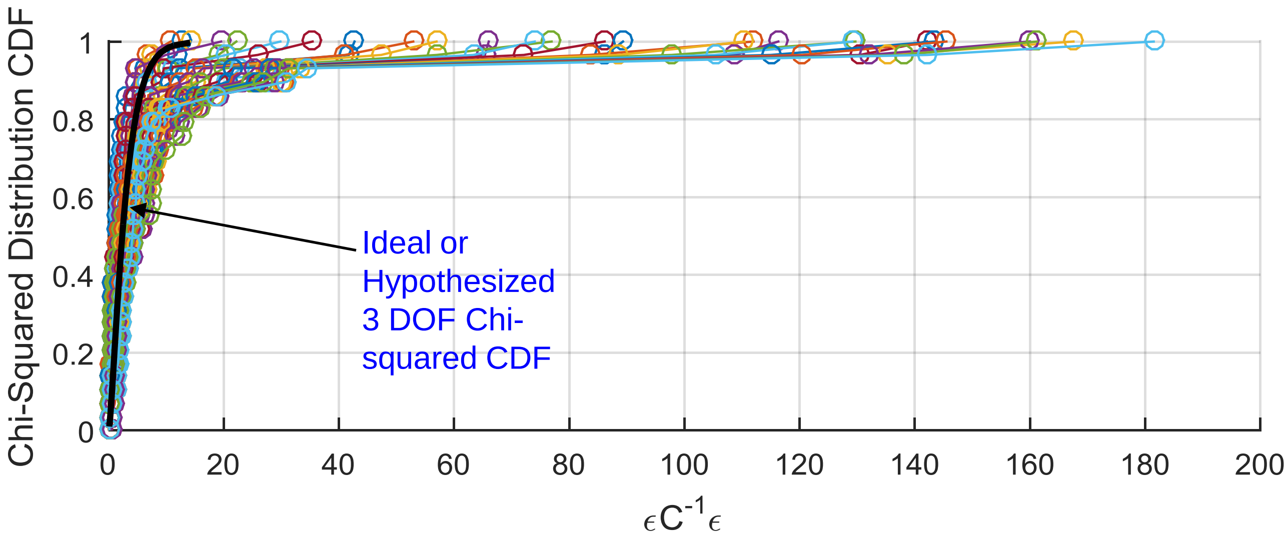


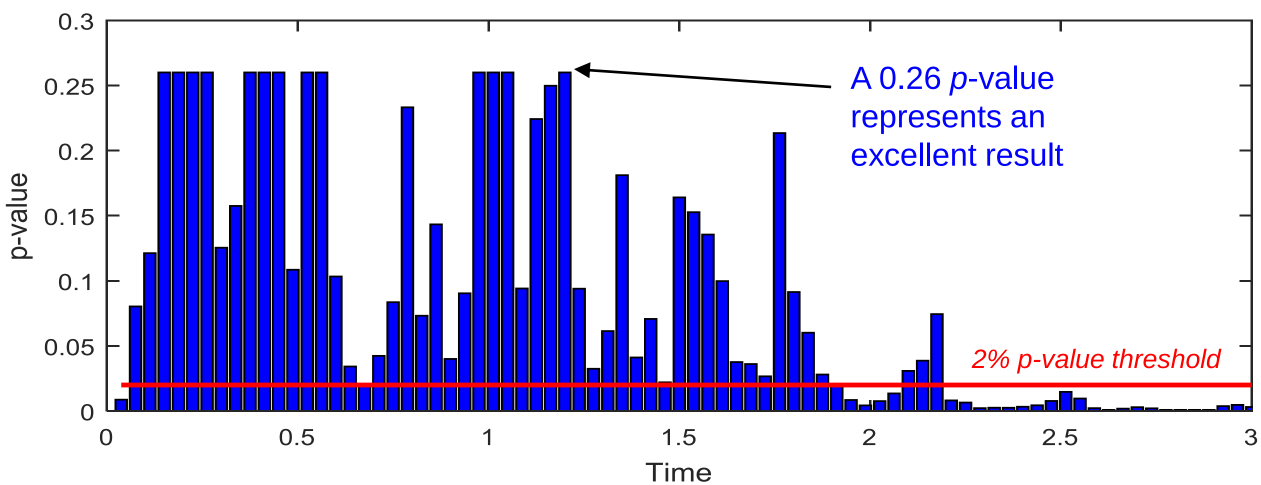
- **Propagation Time Span** – 2 Aug 2014 to 6 Nov 2014
- **Maneuver Dates** – 27 Aug 2014
  17 Sep 2014
  08 Oct 2014
  21 Oct 2014

Mean RMS

Predicted State Estimate Error

Goodness-Of-Fit Results (Without Outlier Identification)



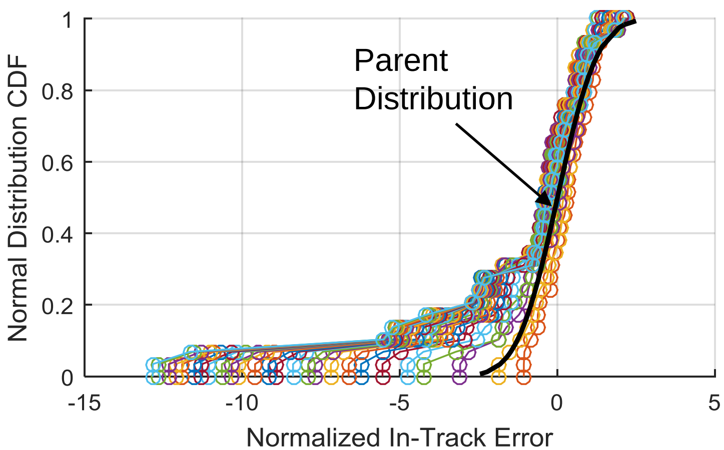


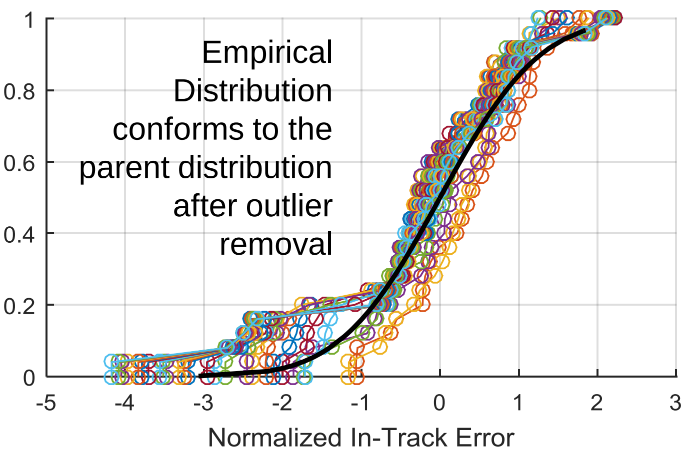
- 80 Bins containing Chi-Square statistics equal to the number of propagations (29) are tested by computing their CDF across the 3-day propagation time span.
- A *p*-value threshold of 0.02 or 2% is set to determine a statistical pass.
- 54 out of 80 Chi-Square Bins (63.75%) produce *p*-value larger than 0.02.
- Statistical failures occur between 2.2 and 3 days in the propagation time span.
- Heavy upper-tail distribution implies covariance is undersized.

Rosner Outlier Test

- Identify the following potential outliers based on the Normalized In-Track Error at the end (largest disparity in error) of each 3 day propagation:
  - 13 Sep 2014
  - 28 Sep 2014
  - 22 Oct 2014
  - 25 Oct 2014
- Perform the Rosner Outlier Identification test using the preceding normalized in-track error values.
- For a 2% significance level, the outlier test results indicate all 4 propagations are outliers and therefore can be eliminated.

```
OutlierPositions = [24 25 14 18];
SigOut = [0.0353 0 0.0775 0];
```





Position	Start Date	$(\epsilon_i-\mu_i)/\sigma_i$
1	2-Aug-14	0.631
2	5-Aug-14	0.671
3	8-Aug-14	0.250
4	11-Aug-14	0.396
5	14-Aug-14	0.164
6	17-Aug-14	0.276
7	20-Aug-14	0.456
8	23-Aug-14	0.497
9	29-Aug-14	0.637
10	1-Sep-14	0.180
11	4-Sep-14	0.263
12	7-Sep-14	-0.261
13	10-Sep-14	0.335
14	13-Sep-14	-1.159
15	19-Sep-14	0.910
16	22-Sep-14	0.638
17	25-Sep-14	-0.367
18	28-Sep-14	-1.078
19	1-Oct-14	0.425
20	4-Oct-14	0.987
21	10-Oct-14	0.395
22	13-Oct-14	0.723
23	16-Oct-14	-0.308
24	22-Oct-14	-3.220
25	25-Oct-14	-2.889
26	28-Oct-14	-0.781
27	31-Oct-14	0.289
28	3-Nov-14	0.594
20	6-Nov-14	0.347

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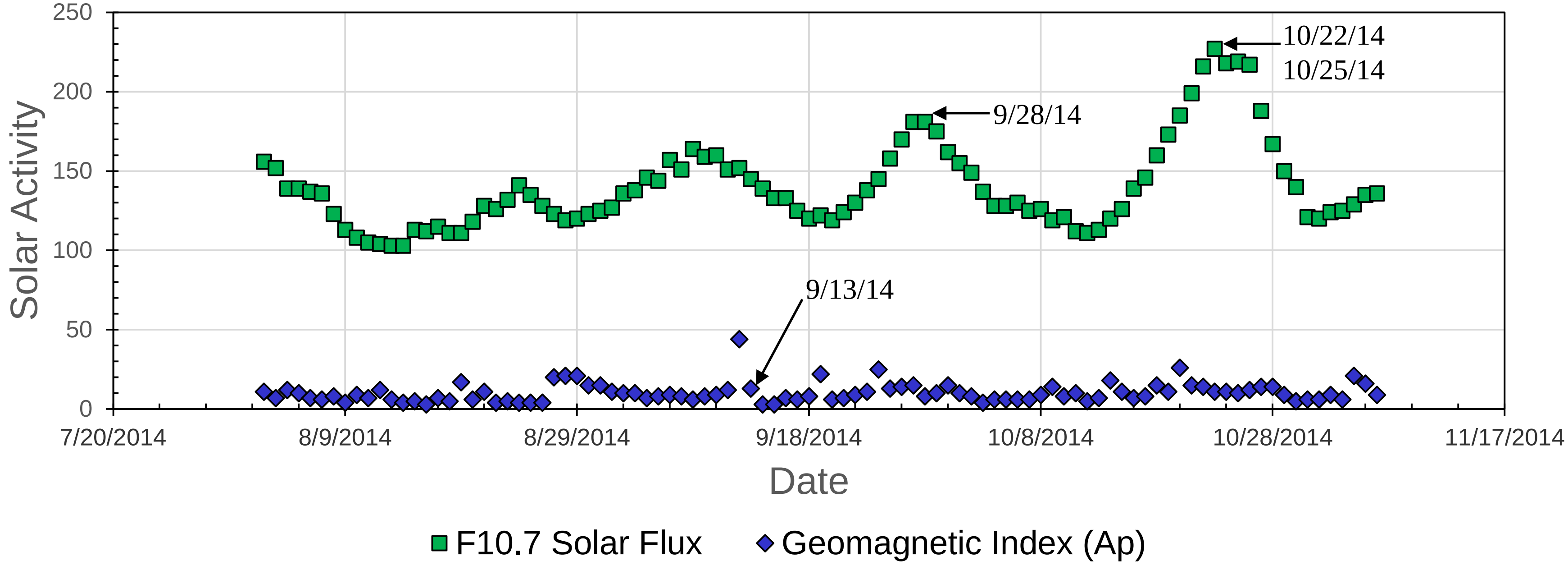
Outlier Rejection Correlation to Solar Activity



- The Outliers identified by the Rosner test show a direct correlation to solar activity on those dates.
- At this time, it appears FDS Propagation is not equipped to predict persistently high solar activity or a dramatic drop in the geomagnetic index.

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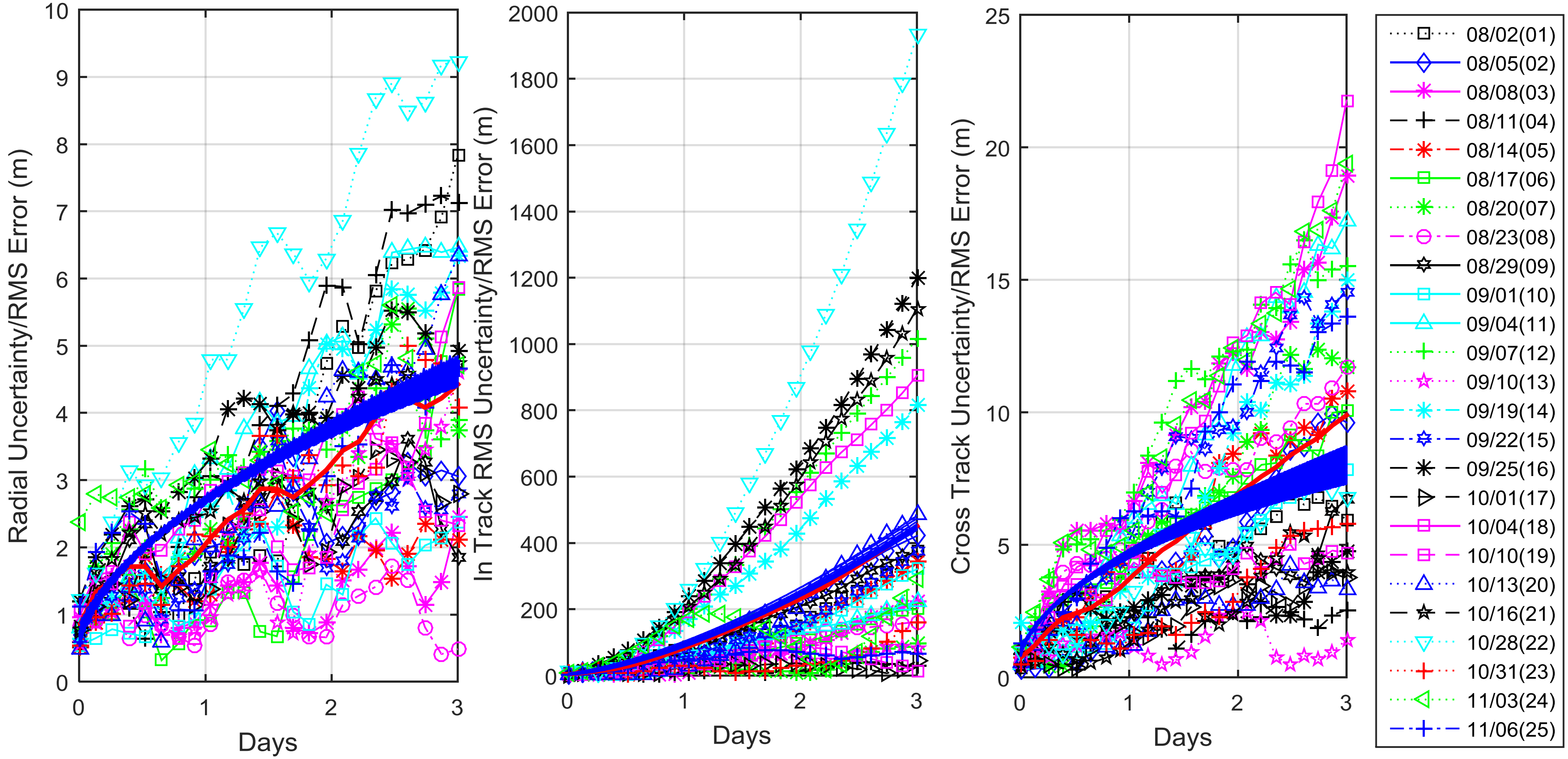
RMS Error and Uncertainty
(With Outlier Identification)

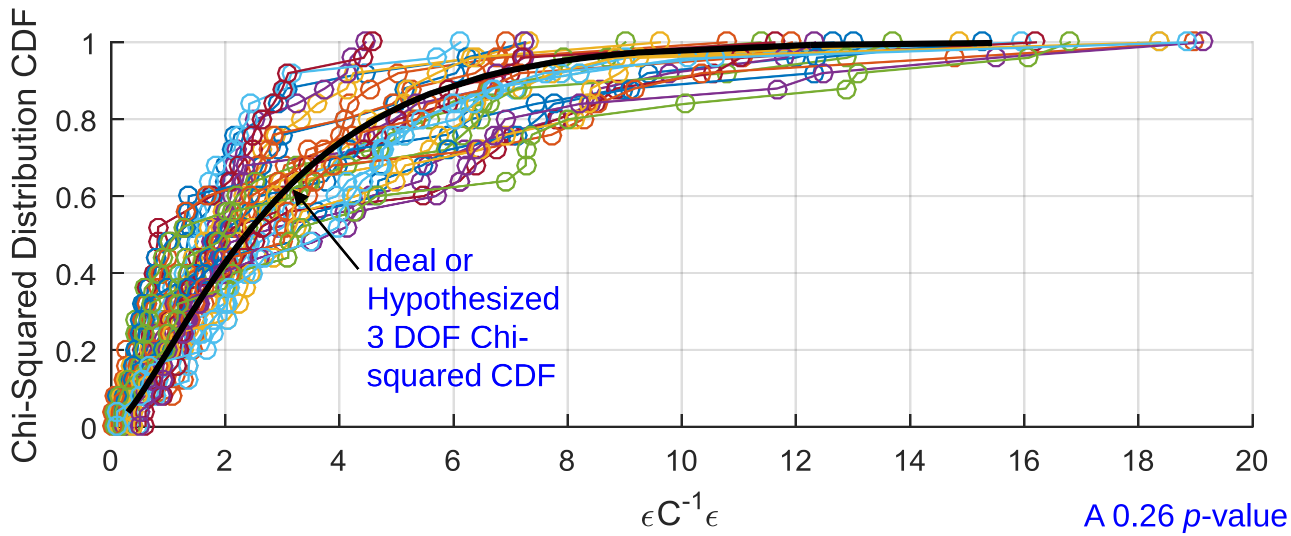


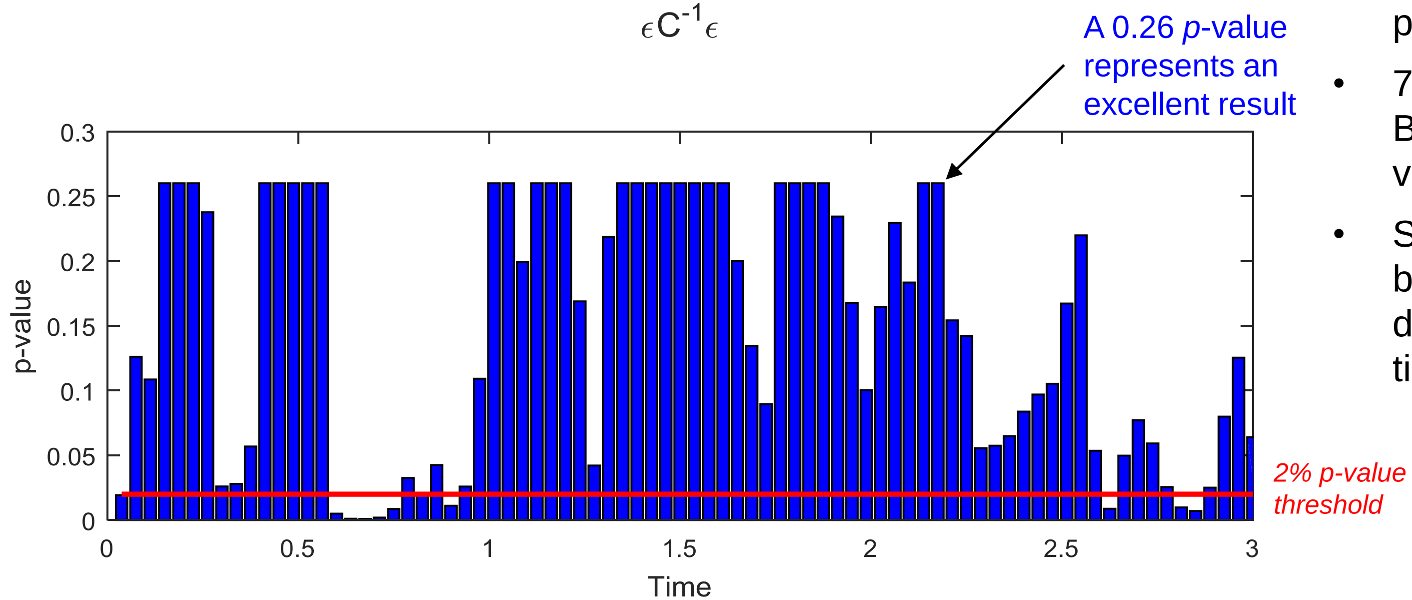
- **Propagation Time Span** – 02 Aug 2014 to 06 Nov 2014
- **Outliers** – 13 Sep 2014 28 Sep 2014 22 Oct 2014 25 Oct 2014
- **Maneuver Dates** – 27 Aug 2014, 17 Sep 2014, 08 Oct 2014, 21 Oct 2014

Mean RMS

Predicted State Estimate Error

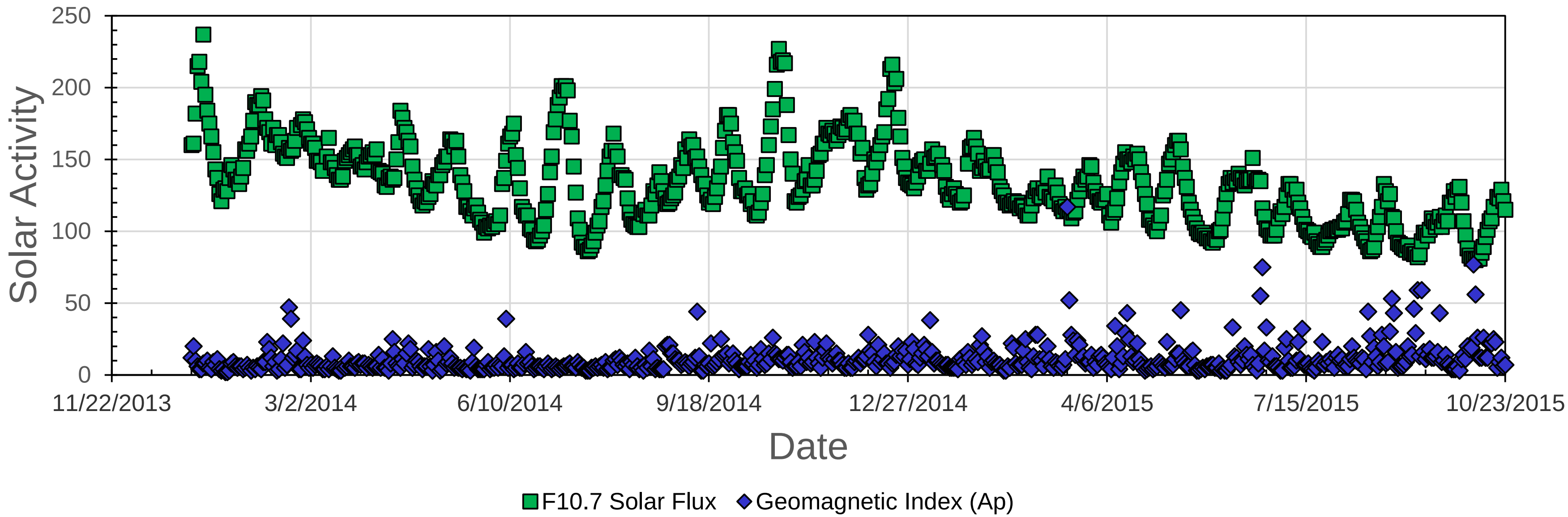
Goodness-Of-Fit Results (With Outlier Identification)

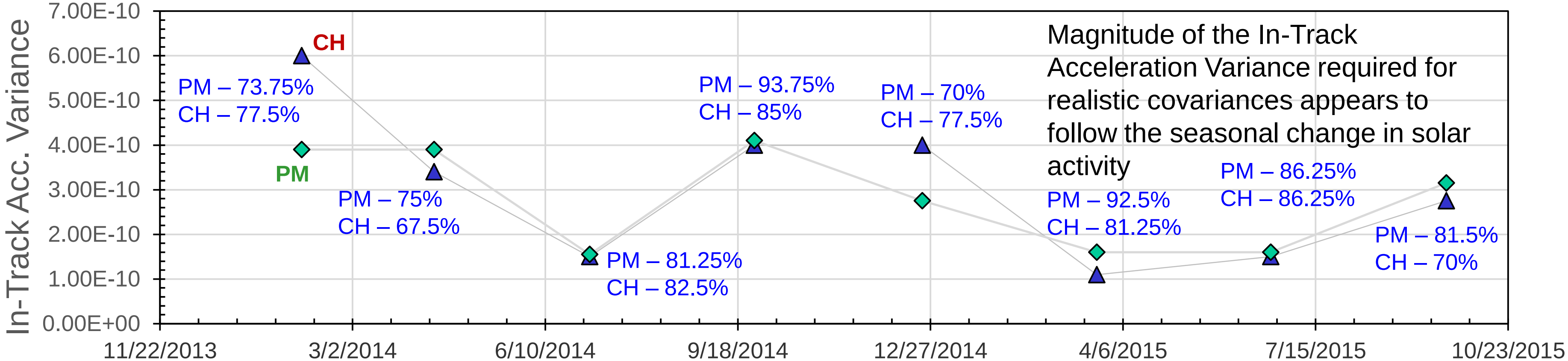




- 80 Bins containing Chi-Square statistics equal to the number of propagations (25) are tested by computing their CDF across the 3-day propagation time span.
- A *p*-value threshold of 0.02 or 2% is set to determine a statistical pass.
- 70 out of 80 Chi-Square Bins (87.50%) produce *p*-value larger than 0.02.
- Statistical failures occur between 0.6 and 0.75 days in the propagation time span.

Aqua and Aura Seasonal Covariance Tuning
Jan 2014 – Oct 2015





Percentage of Chi-Square Statistics above 2%

Impact of a Realistic Covariance on the Probability of Collision (P_C)

- Selected several conjunctions with similar primary and secondary object uncertainties (a rare occurrence) 2.5 – 3 days to TCA.
- Replaced the OCM ASSET covariance with a Tuned O/O covariance.
- Kept miss distance equal to the OCM ASSET solution.
- Examined the changes in uncertainties and their impact on the P_C.

Time of Closest Approach (UTC)	OCM Creation Time (UTC)	OCM Age at TCA (Days)	O/O Cov. Start Date (UTC)	O/O Cov. Age at TCA (Days)	Radial Miss (m)	In-Track Miss (m)	Cross-Track Miss (m)	O/O Radial Cov (m)	O/O In-Track Cov (m)	O/O Cross-Track Cov (m)	OCM ASSET Radial Cov (m)	OCM ASSET In-Track Cov (m)	OCM ASSET Cross-Track Cov (m)	Sec, Object Radial Cov (m)	Sec, Object In-Track Cov (m)	Sec, Object (Cross-Track) Cov (m)	Pc w/ O/O Cov.	Pc w/ OCM ASSET Cov.
03/26/14 03:45:13	03/23/14 03:02:57	3.03	03/23/14 12:00:00	2.66	94.1	2807.6	-1771	5.0	518.2	8.6	13.2	1303.5	3.4	5.3	836	6.8	1.0E-19	1.0E-14
04/07/14 00:22:40	04/04/14 03:27:10	2.87	04/04/14 12:00:00	2.52	-299.3	9766.4	2619.4	4.5	546.7	7.3	8.8	942.3	6.9	10.9	292	16.7	1.3E-71	1.1E-87
12/22/14 18:32:31	12/20/14 03:10:30	2.64	12/20/14 12:00:00	2.27	380.6	-2325.2	-1989.9	4.9	513.6	6.9	17.4	755.4	4.4	13.4	526	7.4	4.9E-67	2.2E-63
03/22/15 15:12:10	03/20/15 02:51:29	2.51	03/20/15 12:00:00	2.13	416.6	-3288.1	976.9	4.9	434	7.9	39.0	647.2	21.0	17.7	380	13.3	5.9E-119	9.0E-42
05/11/15 14:08:45	05/08/15 14:28:10	2.99	05/08/15 12:00:00	3.09	-33.9	984	994.6	4.2	246.4	7.9	10.0	427.4	2.8	8.3	153	6.5	4.7E-12	1.4E-06
05/11/15 14:08:45	05/08/15 17:37:35	2.85	05/08/15 12:00:00	3.09	-29.4	1020.9	1029.4	4.2	246.4	7.9	9.6	394.7	2.8	8.3	153	6.5	6.4E-13	2.4E-07
05/11/15 14:08:45	05/09/15 01:00:08	2.55	05/08/15 12:00:00	3.09	-26.3	995.9	1002.8	4.2	246.4	7.9	9.8	321.6	2.8	7.2	116	6.2	1.1E-13	1.6E-09

Conclusion & Future Work

Conclusion:

- Aqua and Aura are ready to start delivering tuned covariances.
- Covariance realism tuning is sensitive to outliers but can be tuned for up to 3 months at a time.
- Provided the primary and secondary object uncertainties are similar, an impact on the Pc is evident – similar uncertainties involve well-tracked secondary objects.

Future Work:

- Interpolate definitive state estimates and add them to prediction – definitive state estimate.
- Resampling Investigation – Take 1,000 random subsets of a passed Chi-Square Bin and determine the p-values of each test.
- Complete analysis for covariance propagation through maneuvers
- Complete analysis for Terra and GPM.
- Test tuned results with 7-day propagations.
- Group together outliers during high solar events and determine if they conform to a Gaussian distribution – Look at the possibility of increasing process noise during high solar events to more accurately model the predicted state estimate error.

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